

# Supersymmetry Breaking at Finite Temperature in a Susy Harmonic Oscillator with Interaction.

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The supersymmetry breaking in a supersymmetric harmonic oscillator from interaction and at finite temperature is analyzed. Due to the additivity of the thermal effects, the restored supersymmetry through this polynomial interaction, results in nonvanishing energy at finite temperatures while at  $T = 0$  the energy is zero.

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## I. INTRODUCTION

The Supersymmetry has different characteristics than other internal symmetries. One of these characteristics is that in contrast with the internal symmetries, the supersymmetry can be broken for theories with finite fermionic and bosonic degrees of freedom [1, 2]. Therefore we can think of spontaneous symmetry breaking in supersymmetric quantum mechanics. The interest in supersymmetry breaking in quantum mechanics [3]-[4] becomes obvious, when we verify that some results from the breaking of supersymmetric quantum mechanics can be generalized and applied to quantum field theory. In the limit of low energy, the underlying field theory should approach a Galilean invariant supersymmetric field theory [3, 5] and, by the Bergmann superselection rules [6], such a field theory would be equivalent to a supersymmetric Schrodinger equation in each particle number sector of the theory.

Supersymmetry at finite temperature has been studied by [7], [8], [9], [10], [11] and [12]. Nevertheless, the issue of whether supersymmetry is broken at finite temperature even when unbroken at  $T = 0$ . In response to this, [8], suggested that when a change of an operator under SUSY transformation at finite temperature is considered, one should take into account the Klein operator. When this operator is incorporated, [8] shows that the thermal average of this change of an operator is zero for all  $T$ , thereby maintaining supersymmetry at finite temperature. On the other hand, has considered this issue within the context of Thermo Field Dynamics (TFD) [11, 13] and concluded that SUSY is broken at finite temperature, by evaluating the statistical average

of the SUSY Hamiltonian at  $T = 0$  as its vacuum expectation value in the 'thermal vacuum  $|0(\beta)\rangle$  (where  $\beta = 1/kT$ ,  $k$  being the Boltzmann constant) and showing that it is non-zero at finite temperature.

In this paper following TFD, in  $T = 0$  we show that for some conditions the supersymmetric harmonic oscillator is broken getting a bosonic and fermionic harmonic oscillator, with  $\omega_1$  and  $\omega_2$  respectively the bosonic and fermionic frequencies, and the opposite way is possible. Later we show that in agreement with the former results, the supersymmetry is broken at finite temperatures.

## II. SUSY BREAKING OF THE FREE SUSY HARMONIC OSCILLATOR AT FINITE TEMPERATURE.

Consider a Hamiltonian [1] in supersymmetric quantum mechanics in the component form

$$H = \frac{1}{2}[p^2 + W^2(x) + \sigma_3 W'(x)]. \quad (1)$$

The boson-boson interaction is represented by the term  $W^2(x)$ , and the boson fermion is represented by  $\sigma_3 W'(x)$ . Both are determined by the same function  $W(x)$ . This property is found in all supersymmetric models.

As shown in [1],  $W(x)$  varies from the superpotential  $V(x)$ , where  $W(x) = V'(x)$  and  $V'(x)$  is the derivative of  $V(x)$ . Sometimes  $W(x)$  is also called the superpotential.

The double degeneration that occurs in all levels of energy with  $E > 0$ , follows directly from the supersymmetric algebra; it does not depend on  $W(x)$ . In addition we find that the energy of the ground state is non-negative. Similarly this follows from the supersymmetric algebra and is independent of the function  $W(x)$ .

Take  $G_S$  to be the generator of any internal symmetry  $S$ . The symmetry  $S$  is exact and is not spontaneously broken if  $[H, G_S] = 0$  and the ground state  $|0\rangle$  is invariant,  $G_S|0\rangle = 0$ . In the case of supersymmetry, an

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example with exact supersymmetry is the free supersymmetric harmonic oscillator, where  $W = \omega_1 x$  [14, 15].

In terms of the creation and annihilation operators the Hamiltonian becomes,

$$H = \omega_1(a^\dagger a + b^\dagger b) \quad (2)$$

where  $a^\dagger$  and  $b^\dagger$  are respectively the bosonic and fermionic creation operators and the dual  $a$  and  $b$ , are respectively the annihilation operators. The algebra is:

$$a^\dagger \wedge a = -1/2; \quad b^\dagger \bullet b = 1/2$$

where,  $\wedge$  and  $\bullet$  are related the commutator and anticommutator respectively. Following the algorithm of TFD, we double the Hilbert Space [16] writing,

$$\hat{H} = \omega_1(a^\dagger a + b^\dagger b) - \omega_1(\tilde{a}^\dagger \tilde{a} + \tilde{b}^\dagger \tilde{b}) \quad (3)$$

Then we introduce a thermal vacuum so that the statistical average of any operator is given in the thermal vacuum. Then all Feynman techniques of zero temperature field theory can be used.

The thermal energy given in [9], shows that the thermal vacuum  $|0(\beta)\rangle$  of the free supersymmetric harmonic oscillator has non-vanishing energy for a positive temperature. In addition the interaction will not lead to null energy, thus restoring the supersymmetry.

### III. SUPERSYMMETRIC OSCILLATOR WITH INTERACTION

Consider a general Hamiltonian defined by:

$$H = F(\hat{a}, b, \hat{b})a + G(\hat{a}, a, \hat{b})b \quad (4)$$

where  $\hat{a}, a$ , are bosonic fields and  $b, \hat{b}$ , are fermionic fields, and  $F$  is a bosonic polynomial of  $\hat{a}, b, \hat{b}$  and  $G$  is a fermionic polynomial of  $\hat{a}, a, \hat{b}$ . The following transformation preserves the structure of the Hamiltonian,

$$a_2 = a + \beta_1 \hat{b}b; \quad \hat{a}_2 = \hat{a} + \beta_1 \hat{b}b; \quad (5)$$

$$b_2 = (\exp[\beta_2(\hat{a} - a)])b; \quad \hat{b}_2 = \hat{b}(\exp[\beta_2(a - \hat{a})]); \quad (6)$$

where  $\beta_1$  and  $\beta_2$  are real parameters. This transformation defines the Bogoliubov transformation.

Restricting the model, eq.(4), in order to consider a oscillator model with interactions, we define the polynomials  $F = \omega_1 \hat{a}$  and  $G = \omega_2 \hat{b} - \alpha_1 \hat{a} \hat{b} + \alpha_2 a \hat{b}$  where  $\omega_1, \omega_2, \alpha_1, \alpha_2$  are real parameters and the definition of perpendicularity and parallelism in the extended Fock space came from and should read as

$$\begin{aligned} \hat{a} \wedge a &= -1/2; & \hat{a} \bullet a &= n_b + 1/2 \\ \hat{b} \bullet b &= 1/2; & \hat{b} \wedge b &= n_f - 1/2 \end{aligned} \quad (7)$$

$n_b \in \mathbb{N}$  and  $n_f \in \{0, 1\}$ . Requiring that this algebra be invariant by duality, we define:  $\hat{a} = a^\dagger$  and  $\hat{b} = b^\dagger$ .

It is possible to establish some conditions over the oscillator  $H = F(a^\dagger, b, b^\dagger)a + G(a^\dagger, a, b^\dagger)b$ , in the way that it will be a supersymmetric oscillator. To accomplish this we define  $\alpha_1 = -\alpha_2$ ;  $\omega_2 = \frac{(\omega_1)^2 + (\alpha_2)^2}{\omega_1}$ , turning  $H$  a supersymmetric oscillator, that will produce the supercharges and the supersymmetric transformations.

#### A. The Supercharge and Supersymmetric Transformations

The supercharge follows from the condition  $[H, G_S] = 0$ . The supersymmetric oscillator with interactions

$$H = F(a^\dagger, b, b^\dagger)a + G(a^\dagger, a, b^\dagger)b; \quad (8)$$

has the following supercharges  $G_S = a^\dagger b \exp \frac{1}{\omega_1}(\alpha_2 a^\dagger - \alpha_2 a)$  and  $G_S^\dagger = [\exp -\frac{1}{\omega_1} \times (\alpha_2 a^\dagger - \alpha_2 a)]b^\dagger a$ . The supersymmetric transformations of the component fields are through  $G_S$ :

$$\begin{aligned} \delta_{susy} a &= ([\exp \frac{1}{\omega_1}(\alpha_2 a^\dagger - \alpha_2 a)]b \\ &+ \frac{1}{\omega_1} \alpha_2 a^\dagger [\exp \frac{1}{\omega_1}(\alpha_2 a^\dagger - \alpha_2 a)]b)\epsilon; \\ \delta_{susy} b &= 0; \end{aligned}$$

$$\delta_{susy} a^\dagger = \frac{1}{\omega_1} \alpha_2 a^\dagger b [\exp \frac{1}{\omega_1}(\alpha_2 a^\dagger - \alpha_2 a)]\epsilon;$$

$$\delta_{susy} b^\dagger = \frac{1}{\omega_1} a^\dagger [\exp \frac{1}{\omega_1}(\alpha_2 a^\dagger - \alpha_2 a)]\epsilon;$$

where  $\epsilon$  is a Grassmann parameter. Similarly, transformations from the conjugation above.

The interactions terms follow from the polynomial  $G(a^\dagger, a, b^\dagger)$  giving  $H = H_0 + H_{\text{int}}$  where

$$H_0 = \omega_1 a^\dagger a + \omega_2 b^\dagger b,$$

and

$$H_{\text{int}} = \alpha_2 a^\dagger b^\dagger b + \alpha_2 a b^\dagger b. \quad (9)$$

Though  $H$  is a supersymmetric oscillator,  $H_0$  is not a supersymmetric harmonic oscillator due to the fact that  $\omega_1 \neq \omega_2 = \frac{(\omega_1)^2 + (\alpha_2)^2}{\omega_1}$ .

From the Eq.(5)-Eq.(6), after we perform the Bogoliubov transformation with  $\alpha_1 = -\alpha_2$ , that preserves the algebra, Eq.(7), we obtain the harmonic oscillator with the bosonic and fermionic frequencies  $\omega_1$  and  $\omega_3$ , respectively.

$$H = \omega_1 a^\dagger a + \omega_3 b^\dagger b$$

The condition to be supersymmetric harmonic oscillator is  $\omega_3 = \omega_1$ , that relates to Eq.(9) where:

$$\begin{aligned} \omega_{1\pm} &= \frac{1}{2}(\omega_2 \pm (\omega_2^2 - 4\alpha_2^2)^{1/2}); \\ \omega_2 &= 2\alpha_2 + \xi \end{aligned}$$

$\xi$  parametrize  $\omega_2$  at phase space.

#### IV. SUPERSYMMETRY BREAKING AT FINITE TEMPERATURE FOR THE SUPERSYMMETRIC OSCILLATOR WITH INTERACTION

The supersymmetry will be broken if the thermal vacuum  $|0(\beta)\rangle$ , from the supersymmetric oscillator with interaction Eq.(4) has nonvanishing energy for a positive temperature. To introduce the temperature, we double the Hilbert space following the algorithm of TFD. That will allow us to calculate the thermal vacuum and then the statistical average of the Hamiltonian operator of the supersymmetric oscillator with interaction,

$$H = F(a^\dagger, b, b^\dagger)a + G(a^\dagger, a, b^\dagger)b \quad (10)$$

where:  $F = \omega_1 a^\dagger$  and  $G = \omega_2 b^\dagger + \alpha_2 a^\dagger b^\dagger + \alpha_2 a b^\dagger$ , and  $\omega_2 = \frac{(\omega_1)^2 + (\alpha_2)^2}{\omega_1}$ ;  $\omega_1$  and  $\alpha_2$  are real parameters. With the algebra

$$\begin{aligned} a^\dagger \wedge a &= -1/2; & a^\dagger \bullet a &= n_b + 1/2; \\ b^\dagger \bullet b &= 1/2; & b^\dagger \wedge b &= n_f - 1/2; \end{aligned} \quad (11)$$

$n_b \in \mathbb{N}$  and  $n_f \in \{0, 1\}$ . That will give us the energy.

We follow the TFD program to introduce temperature in the system, and double the boson and fermion creation and annihilation operators, directly from the Hamiltonian Eq.(10). The statistical average of the Hamiltonian operator is

$$\begin{aligned} \hat{H} = H + \tilde{H} &= F(a^\dagger, b, b^\dagger)a + G(a^\dagger, a, b^\dagger)b \\ &+ F(\tilde{a}^\dagger, \tilde{b}, \tilde{b}^\dagger)\tilde{a} + G(\tilde{a}^\dagger, \tilde{a}, \tilde{b}^\dagger)\tilde{b}. \end{aligned}$$

But an elegant way to obtain the results is to use Eq.(8), and the Bogoliubov Transformation that preserves the algebra given in Eq.(11). The Bogoliubov transformations are

$$\begin{aligned} a_2 &= a + \frac{\alpha_2}{\omega_1} b^\dagger b; & a_2^\dagger &= a^\dagger + \frac{\alpha_2}{\omega_1} b^\dagger b; \\ b_2 &= (\exp[\frac{\alpha_2}{\omega_1}(a^\dagger - a)])b; & b_2^\dagger &= b^\dagger (\exp[\frac{\alpha_2}{\omega_1}(a - a^\dagger)]). \end{aligned} \quad (12)$$

Transforming the supersymmetric oscillator, Eq.(10), using the Bogoliubov transformation, Eq.(12), we get

$$H = \omega_1 a_2^\dagger a_2 + \omega_1 b_2^\dagger b_2, \quad (13)$$

the algebra  $a_2^\dagger \wedge a_2 = -1/2$ ;  $a_2^\dagger \bullet a_2 = n_b + 1/2$ ;  $b_2^\dagger \bullet b_2 = 1/2$ ;  $b_2^\dagger \wedge b_2 = n_f - 1/2$ . Showing that the supersymmetric oscillator with interaction Eq.(8) is studied as a supersymmetric harmonic oscillator with the frequency

$\omega_1$ . Now following TFD, we double the Hilbert space getting the states

$$\begin{aligned} |0\rangle &= |0; 0\rangle = |0\rangle \times |0\rangle \\ |n_b, n_f; \tilde{n}_b, \tilde{n}_f\rangle &= |n_b, n_f\rangle \times |\tilde{n}_b, \tilde{n}_f\rangle. \end{aligned}$$

Any operator  $A(\beta)$  and the thermal vacuum  $|0(\beta)\rangle$  at a finite temperature are obtained from the zero temperature respectively by

$$A(\beta) = e^{-iG} A(0) e^{iG}, \quad |0(\beta)\rangle = e^{-iG} |0\rangle,$$

where  $G = -i\theta(\beta)(\tilde{b}b - b^\dagger \tilde{b}^\dagger) - i\theta(\beta)(\tilde{a}a - a^\dagger \tilde{a}^\dagger)$ ,  $\beta = 1/\kappa T$ ,  $\kappa$  is the Boltzmann constant with  $\tan \theta(\beta) = e^{-\beta\omega_1/2}$ .

The thermal energy of the thermal vacuum is given by

$$E_0(\beta) = \langle 0(\beta) | H | 0(\beta) \rangle = \langle 0(\beta) | \omega_1 a_2^\dagger a_2 + \omega_1 b_2^\dagger b_2 | 0(\beta) \rangle =$$

$$\omega_1 \left( \frac{e^{-\beta\omega_1}}{1 - e^{-\beta\omega_1}} + \frac{e^{-\beta\omega_1}}{1 + e^{-\beta\omega_1}} \right). \quad (14)$$

This shows that the supersymmetric is broken at  $T > 0$ , (Fig.1).

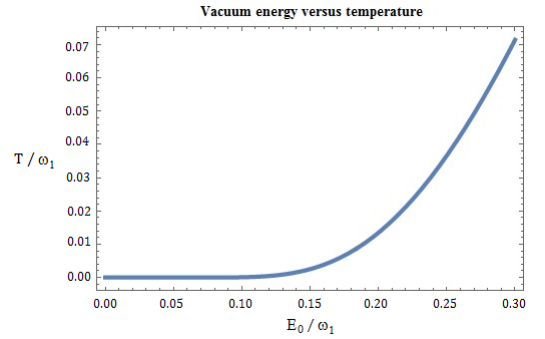


Figure 1. Plot of the vacuum energy  $E_0/\omega_1$ , as function of  $T/\omega_1$ .

The Witten index is:

$$\Delta(\beta) = \frac{1 - e^{-\beta\omega_1}}{1 + e^{-\beta\omega_1}}.$$

The action of the Supersymmetric charges over the thermal vacuum is given by:

$$G_{2S} |0(\beta)\rangle = a_2^\dagger b_2 |0(\beta)\rangle = \frac{e^{-\beta\omega_1/2}}{[(1 - e^{-\beta\omega_1})(1 + e^{-\beta\omega_1})]^{1/2}} |\chi_1(\beta)\rangle, \quad (15)$$

$$\overline{G_{2S}} |0(\beta)\rangle = b_2^\dagger a_2 |0(\beta)\rangle = \frac{e^{-\beta\omega_1/2}}{[(1 - e^{-\beta\omega_1})(1 + e^{-\beta\omega_1})]^{1/2}} |\chi_2(\beta)\rangle, \quad (16)$$

where  $|\chi_1(\beta)\rangle$  and  $|\chi_2(\beta)\rangle$  are the Goldstino states at finite temperature, that are produced from the vacuum by applying supersymmetry charges.

## V. CONCLUSION

An oscillator model with polynomial interactions is analysed and we show conditions that lead the same to a supersymmetric harmonic oscillator with interaction. The temperature is introduced in this oscillator model with interaction, in TFD formalism. Reducing the model to a supersymmetric harmonic oscillator so at finite temperature we analyse the supersymmetry breaking. The fermionic and bosonic quantum corrections in a supersymmetric theory tend to cancel. The thermal effects are additive and the thermal energies are positive. The statistical average of the Hamiltonian at finite temperature is not zero.

At  $T = 0$  from the same parameter  $\alpha_2$  of polynomial

interaction and frequency  $\omega_2 > 2\alpha$  we obtain two bosonic frequencies  $\omega_1$  that gives supersymmetric harmonic oscillator  $(\omega_{1-}, \omega_{1-})$  and  $(\omega_{1+}, \omega_{1+})$ . And for the case  $\omega_2 = 2\alpha_2$ , we have only one supersymmetric harmonic oscillator for the same polynomial parameter of interaction because  $\omega_{1-} = \omega_{1+}$ .

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